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Stationary phase point extraction based on high – resolution radon transform and its application to multiple attenuation by wavefield extrapolation

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Abstract: Multiple prediction and subtraction techniques based on wavefield extrapolation are effective for suppressing multiple related to water layers. In the conventional wavefield extrapolation method, the multiples of the seismic data are predicted from the known total wave field by the Green function convoluted with each point of the bottom. However, only the energy near the stationary phase point has an effect on the summation result when the convolutional gathers are added. The research proposed a stationary phase point extraction method based on high-resolution radon transform. In the radon domain, the energy near the stationary phase point is directly added along the convolutional gathers curve, which is a valid solution to the problem of the unstable phase of the events of multiple. The Curvelet matching subtraction technique is used to remove the multiple, which improved the accuracy of the multiple predicted by the wavefield extrapolation and the artifacts appearing around the events of multiple are well eliminated. The validity and feasibility of the proposed method are verified by the theoretical and practical data example.

Key words: high-resolution radon transform; multiple; wavefield extrapolation; stationary phase point

1 Introduction

Multiple attenuation is one of the most prominent problems in marine seismic exploration. How to effectively suppress multiple and highlight primary is one of the keys in marine seismic data processing (Monk, 1993; Weglein, 2012; Verschuur & Berkhout, 1997; Weglein *et al.*, 1997). At present, the mainstream of multiple suppression technology can be divided into two categories (Verschuur, 1999): the first one is the filtering method based on the signal analysis, which removes multiple by looking for the difference between primary and multiple; the second one is prediction and subtraction method based on the wave equation, which utilizes the wavefield theory that produces multiple, trying to get an accurate multiples model, then the multiple from the original data is subtracted, so as to achieve the suppression of the multiple.

Multiple prediction and subtraction techniques based on wavefield extrapolation belong to the second method. It is an important method to eliminate the surface-related multiple. In view of its simple principle and good application effect, this method has been widely used in recent years. It consists of two steps: the first step is to predict the multiples model from the

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velocity model or seismic data, and the second step is to subtract the multiple from the seismic data by adaptive matching (Wiggins, 1988; Verschuur *et al.*, 1992). In the case of wavefield extrapolation, the multiples of the seismic data are predicted from the known total wave field by the convolution of the Green function with each point of the bottom. However, only the energy near the stationary phase point has an effect on the summation result when the convolutional gathers were added (Berkhout & Verschuur, 2005).

In this paper, the stationary phase point of convolutional gathers in the process of addition is discussed. Based on the idea of superposition of stationary phase point, the method of stationary phase points extraction based on high-resolution radon transform is proposed. In the high-resolution radon domain, the energy near the stationary phase point is directly added along the convolutional gathers curve (Dedem & Verschuur, 2002; Wang, 2007), which improves the accuracy of the predicted multiple and the artifacts appearing around the events of multiple are eliminated. In addition, it provides a good foundation for the follow-up multiple adaptive subtractions. At the same time, the Curvelet matching subtraction technique is used to remove the multiple (Herrmann *et al.*, 2008), which improve the final separation of primary and multiple. By using the method for theoretical and practical data example and comparing with that of conventional wave field extrapolation, it is shown that the method proposed in this paper is more accurate for the prediction of multiple and more obvious for multiple attenuation.

2 Wavefield extrapolation to multiple attenuation

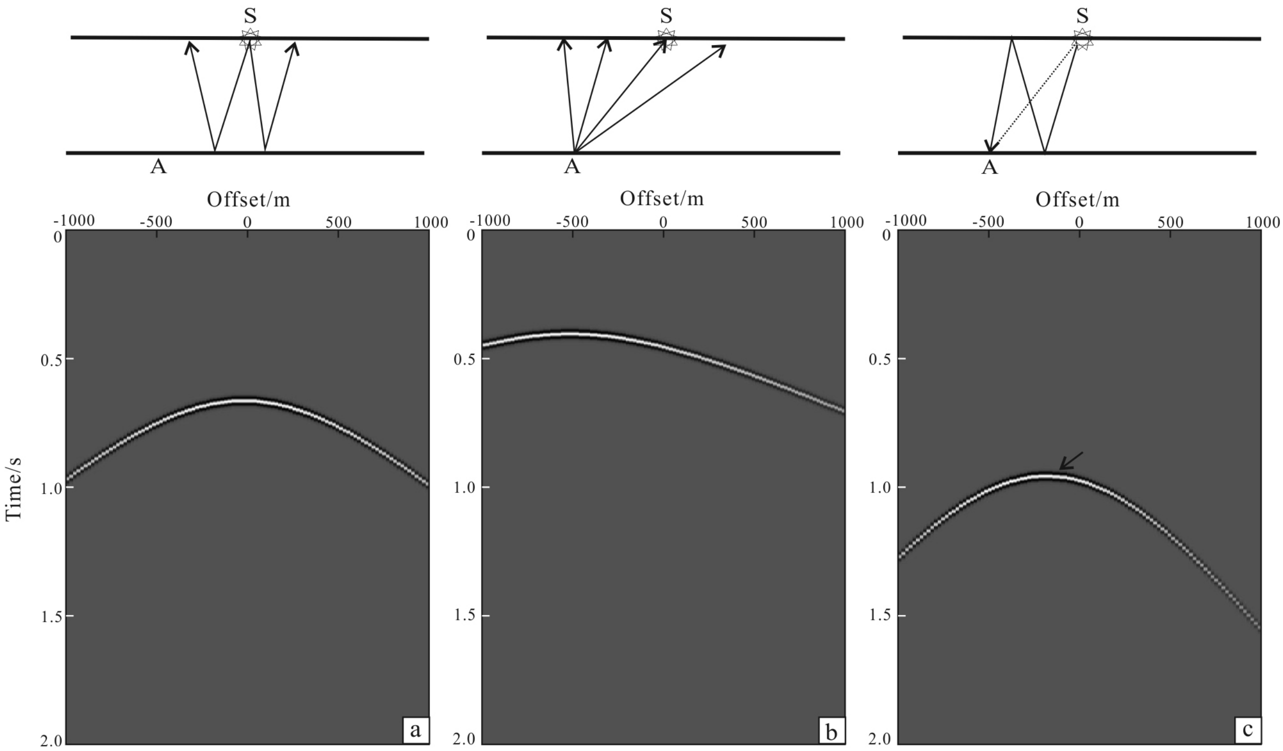
Using the Rayleigh integral II formula:

$$P_A = \frac{1}{2\pi}\int_S P \frac{1 + j2\pi kr}{r^2}\cos\varphi e^{-j2\pi kr} dS \qquad (1)$$

k is the wave number, S is the surface, P_A is the pressure field value of A in the frequency domain, r is the distance from any point on plane S to A. φ is the angle between the vector r and the vertical direction. According to formula (1), in the case of the wavefield on a known plane, the wavefield at point A is obtained by multiplying each point on the plane by a time-shift operator and amplitude factor associated with the wavefield position and summing all results of time-shifted and scaled measurements (Fig. 1). Similarly, the wavefield at the sea bottom can be obtained. Rayleigh integral II can satisfy the need of actual seismic exploration to complete the task of predicting the unknown wavefield according to the known wavefield that is wavefield extrapolation method based on wave equation (Berkhout & Pao, 1982).

Wavefield extrapolation predicting multiple method is to add a seismic wave propagation in the water layer. The order of the multiple of the water layer is increased, so that the first order multiple becomes the second order multiple, the second order multiple becomes the third order multiple, and so on. Firstly, a layered model is established, and the seismic wavefield received on the surface is extended forward to the water bottom according to the extrapolation method and then multiplied by the reflection coefficient R of the reflective layer. Finally, the wavefield extending to the bottom is forward extended to the surface. This is the process of simulating the multiple of the water layer. The surface multiple model predicted in this way has some amplitude and phase differences compared with the actual multiple in the original data. In the subsequent processing step, the predicted multiple model is matched and then subtracted from the original data.

In this paper, Curvelet domain matching subtraction technique is used to subtract the predicted multiple from the original data. The sparsity of Curvelet transform makes it use very few coefficients to simulate the signal, and has a strong anti-interference ability to be affected by the signal to noise ratio. Its directivity makes the Curvelet transform a very accurate representation of the curves for events of multiple. Therefore, Curvelet transform has a very good effect on the matching of multiple. The advantages of Curvelet threshold subtraction method are that it is able to adapt to the deviation of multiple matching processing, and is better in suppressing the boundary effects and waveform distortion, so that there are very continuous events of multiple.



(a) The common shot gathers; (b) the Green's function; (c) the convolution of (a) and (b) in the time direction.

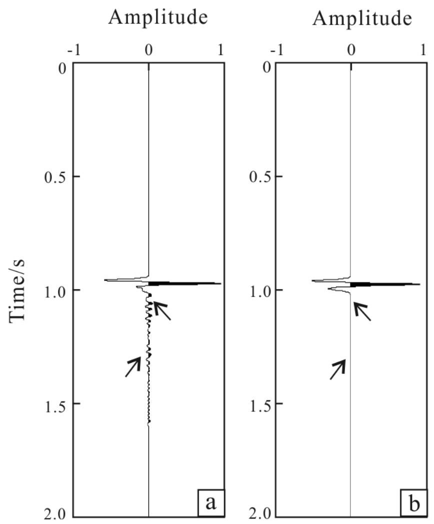
Fig. 1 wavefield extrapolation

3 Stationary phase point extraction based on high-resolution radon transform

Fig. 1 clearly illustrates that in the process of the wavefield extrapolation; firstly, the common-shot gathers and the Green's functions at each point of the ocean bottom are convoluted in time direction to form convolutional gathers. Then, the sum of the convolutional gathers added. It is noted that in all convolutional gathers, it is only advantageous to construct a multiple prediction path by the gathers obtained from the convolution consistent with Snell's law. That is, only the energy near the stationary phase point has an effect on the summation (arrow in Fig. 1c). However, if the convolutional gathers is directly superimposed, it will bring a lot of artifacts or noise interference to the result, which will interfere with the prediction of events of multiple and multiple matching subtraction, and reduce the signal-to-noise ratio of the seismic record (Fig. 2a).

For the above reasons, this research studied a new method for superposition of convolutional gathers. Due to the paraxial curve near the stationary phase point of the multiple convolutional gather is approximately a parabola; the high-resolution radon transform can well describe the nonlinear events near the stationary phase point of the convolutional gathers. Therefore, a stationary phase point extraction method based on high-resolution radon transform is proposed to predict the multiples directly from the convolutional gathers formed by the seismic records and the Green's functions.

Firstly, the convolutional gathers of the common shot gathers and Green's functions is transformed from the space-time domain to the parabolic radon domain



(a) The common shot gathers; (b) the Green's function; (c) the convolution of (a) and (b) in the time direction.

Fig. 2 Amplitude comparison of direct superposition and high-resolution radon transform stationary phase point extraction superposition

by using the parabolic radon transformation. The formula is:

$$R_{p\tau}\{d(x,t)\} = \int \int_{-\infty}^{\infty} d(x,t)\,\delta[t - (\tau + px^2)]\,dtdx = m(p,\tau) \quad (2)$$

$d(x,t)$ is data field, $m(p,\tau)$ is the radon domain, p is the curvature of the parabola. The transform expressed in the form of a matrix $m = Ld$. The conjugate matrix L^T of operator L is defined as its approximate inverse. The inverse transform can be expressed in the form of a matrix $d = L^T m$.

Firstly, the inverse transformation from the radon domain to the space-time domain is defined. Then, an estimation of the least squares inverse of the model operator m. Then there is use of the sparse constrained preconditioned conjugate gradient method to solve the least squares inverse problem, which can achieve high-resolution radon transform. Use the damping factor $\mu = \frac{\sigma_n^2}{\sigma_m^2}$, P is a diagonal matrix. The elements on its diagonal are:

$$P_{ii} = 1 + \frac{m_i^2}{\sigma_m^2}, i = 1, \ldots N_m \quad (3)$$

The least squares inverse of L^T at this time can be expressed as:

$$(L^T)^{-1} = (L \times L^T + \mu P^{-1})^{-1} L \quad (4)$$

The positive transformation from the space-time domain to the radon domain can be expressed as:

$$m = (L^H L + \mu P)^{-1} L^H d \quad (5)$$

The above equation is a nonlinear inverse problem and can be solved by nonlinear conjugate gradient optimization algorithm. The solution of this nonlinear inverse problem is controlled by σ_n and σ_m. σ_n denotes an estimate of the Gaussian noise in the data, which includes the random noise in the data and the noise caused by the uncertainty in the process of defining the positive transformation operator L. σ_m controls the degree of sparsity in the solution. The stability of the inversion is controlled by using the damping factor μ. In practice, the fixed value of σ_n is generally taken, and the effect of the inversion is adjusted by using σ_m.

In the radon domain, the convolutional gathers is directly summed. The formula is:

$$M(t_r, x_r) = \sum_{q=q_{\min}}^{q=q_{\max}} m(q,\tau) \quad (6)$$

First, the inversion is performed by a parametric model. Then, a radon transformation is performed. Finally, the summation can be used to complete stationary phase point extraction based on high-resolution radon transform and thus, obtain the predicted multiple of water layers.

Fig. 2 is amplitude comparison of the direct superposition and high-resolution radon transform stationary phase point extraction superposition. Direct superposition will bring great artifacts, which disturbed the phase identification of the events. After the superposition of the radon domain, the energy near the stationary phase point is strengthened, and the artifacts information is well restrained.

The multiple of water layers predicted in this way has a certain amplitude and phase difference compared with the actual multiple in the original data. In the subsequent processing step, the adaptive matching and subtraction will be taken.

4 Numerical example

In order to verify the effectiveness of the proposed method, first a simple layered model is experimentally processed and analyzed. The model consists of three layers of horizontally layered media (Fig. 3). The depth of the first layer is 400 m and the velocity is 1 500 m/s. Based on staggered grids the differenti–

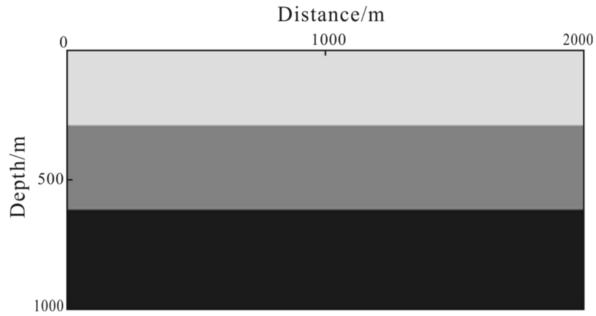


Fig. 3 Underground medium model

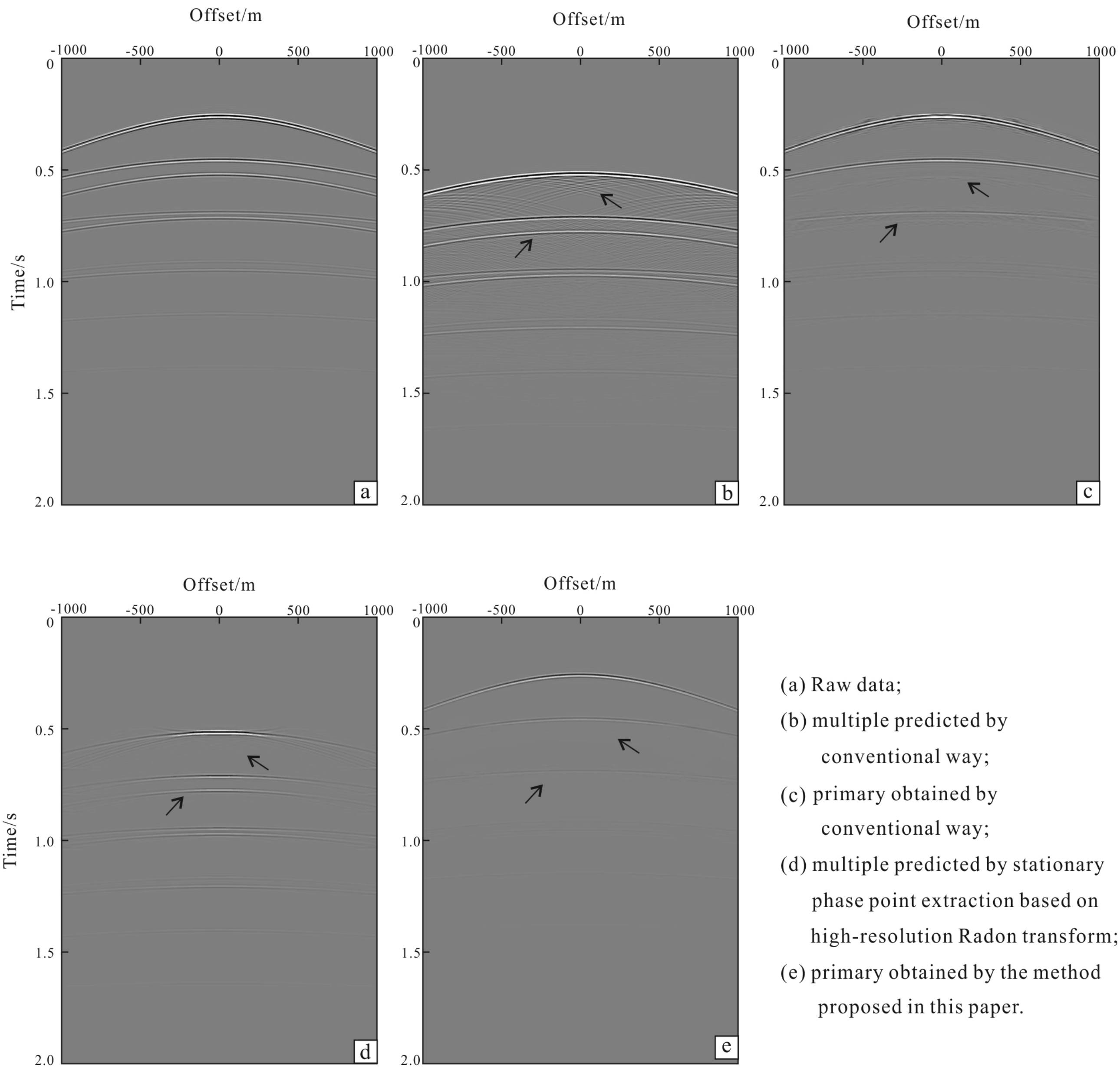


Fig. 4 Results of numerical examples

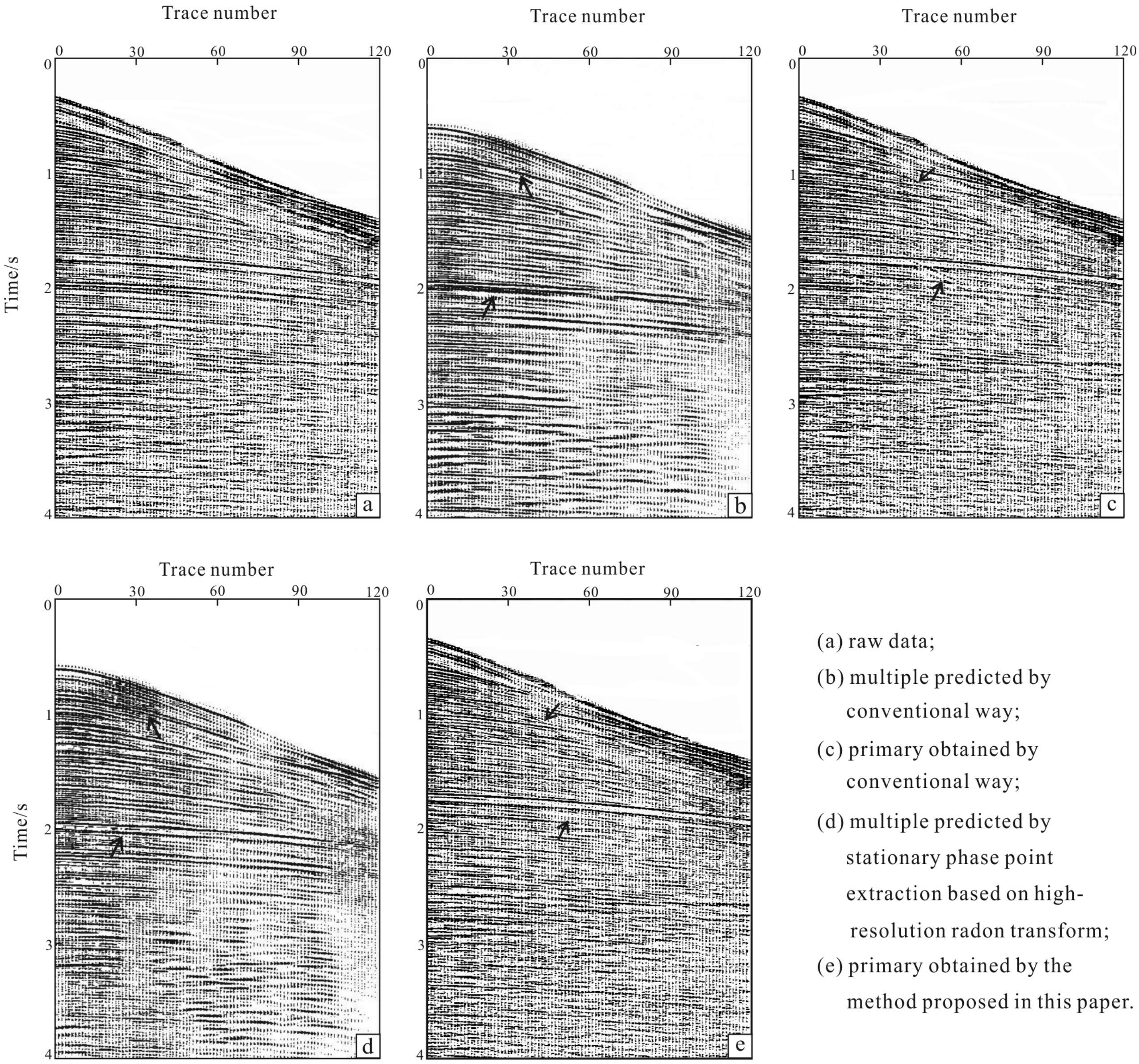


Fig. 5 Results of practical data examples

the model data is processed by multiple prediction and subtraction techniques based on wavefield extrapolation.

Fig. 4a is the raw data, in which most of the multiple are generated in the water layer; Fig. 4b is the multiple predicted by the direct superposition; Fig. 4c is the primary obtained by subtracting the multiple of Fig. 4b from Fig. 4a by Curvelet matching; Fig. 4d is the multiple predicted by stationary phase point extraction based on high-resolution radon transform; and Fig. 4e is the primary obtained by subtracting the multiple of Fig. 4d from Fig. 4a by Curvelet matching.

Comparing Fig. 4b with Fig. 4d, illustrates that the conventional superposition of convolutional gathers and the method proposed in this paper, both can predict multiple. However, with the multiple by the first way will appear artifacts or noise, which disturbed the phase identification of the events. After the superposition of the high-resolution radon domain, the energy near the stationary phase point is strengthened, and the artifacts information is well restrained. The events of multiple are clearer and the phase is very close to the real phase.

Comparing Fig. 4c with Fig. 4e, shows the primary obtained by the two methods, while the remaining multiple of water layers will appear in the conventional method (Fig. 4c). This is because the multiple artifacts information predicted in Fig. 4b greatly disturbs the adaptive matching subtraction phase, so that the multiple are not completely removed. The high-resolution radon transform method improves the resolution and precision of the multiple, which is more favorable for the multiple matching subtraction so as to get a better multiple attenuation effect (Fig. 4e).

5 Practical data example

A 2D survey data of Bohai Sea Area A was selected as an example. Work area water depth is 230—300 m, and the seabed is relatively flat, so the multiple of water layers is well developed. This research used the conventional direct superposition and stationary phase point extraction based on high-resolution radon transform method to predict the multiples of the data by wavefield extrapolation. Then the Curvelet matching subtraction technique was used to remove the predicted multiple.

Fig. 5a is the original shot record containing multiple. Fig. 5b and Fig. 5d are the multiple predicted by the conventional wavefield extrapolation method and the method proposed in this paper, respectively. It illustrates that, the events of multiple in Fig. 5d are clearer and the artifacts information is well restrained.

Fig. 5c and Fig. 5e are the primary obtained by Curvelet matching which is used to subtract the multiples predicted in Fig. 5b and Fig. 5d, respectively. It shows that there are still some multiple of water layers remaining in Fig. 5c. In Fig. 5e, the multiple of the water layer are well suppressed, and the events of the primary are more obvious.

6 Conclusion

In this paper, there was a discussion of the stationary phase point extraction of convolutional gathers in the process of multiple attenuation by the wavefield extrapolation and use of the sparse constrained preconditioned conjugate gradient method to solve the least squares inversion problem, through which can be achieved high-resolution radon transform. A method of stationary phase point extraction based on high-resolution radon transform is proposed. Then the superposition of the convolutional gathers is directly added in the radon domain to obtain the multiple predicted with higher accuracy and resolution. At the same time, Curvelet transform matching subtraction method with good anisotropy and direction discrimination ability is adopted in the matching subtraction phase so as to obtain the data which the multiple of the water layer have been eliminated. Theoretical data and practical seismic data example show that the method proposed in this paper achieves a better result, that the phase of the multiple predicted by wavefield extrapolation is more accurate and also suppresses the artifacts. The events of the primary are clearer after the multiple of water layers attenuation.

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